# MA114 Summer 2018
## Worksheet 23 – Calculus with Parametric Equations – 7/24/18

1. For the following parametric curves, find an equation for the tangent to the curve at the specified value of the parameter.

   (a) $x = e^{\sqrt{t}}, y = t - \ln(t^2)$ at $t = 1$

   (b) $x = \cos(\mu) + \sin(2\mu), y = \cos(\mu)$ at $\mu = \pi/2$.

2. For each parametric curve, find $dy/dx$:

   (a) $x = e^{\sqrt{s}}, y = s + e^{-s}$

   (b) $x = t^3 - 12t, y = t^2 - 1$

   (c) $x = 4\cos(\omega), y = \sin(2\omega)$

3. Find $d^2y/dx^2$ for the curve $x = 7 + t^2 + e^t, y = \cos(t) + \frac{1}{t}, 0 \le t \le \pi$.

4. Find the arc length of the following curves:

   (a) $x = 1 + 3t^2, y = 4 + 2t^3, 0 \le t \le 1$.

   (b) $x = 4\cos(\theta), y = 4\sin(\theta), 0 \le \theta \le 2\pi$.

   (c) $x = 3v^2, y = 4v^3, 1 \le v \le 3$.

1) a) $\frac{dx}{dt} = \frac{1}{2\sqrt{t}} e^{\sqrt{t}}$, $\frac{dy}{dt} = 1 - \frac{2}{t}$. At $t=1$: $\frac{dx}{dt} = \frac{e}{2}$, $\frac{dy}{dt} = -1$, so $\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = -\frac{2}{e}$

$x(1) = e$, $y(1) = 1$, so $\boxed{y - 1 = -\frac{2}{e}(x-e)}$ is the tangent.

b) $\frac{dx}{d\mu} = -\sin(\mu) + 2\cos(2\mu)$, $\frac{dy}{d\mu} = -\sin(\mu)$, At $\mu = \frac{\pi}{2}$, $\frac{dx}{d\mu} = -1 + -2 = -3$

$\frac{dy}{d\mu} = -1$

So $\frac{dy}{dx} = \frac{-1}{-3} = \frac{1}{3}$. $x(\frac{\pi}{2}) = 0$, $y(\frac{\pi}{2}) = 0$,

so $\boxed{y = \frac{1}{3}x}$ is the tangent line.

2) a) $\frac{dx}{ds} = \frac{1}{2\sqrt{s}} e^{\sqrt{s}}$, $\frac{dy}{ds} = 1 - e^{-s}$, $\frac{dy}{dx} = \frac{dy/ds}{dx/ds} = \frac{1 - e^{-s}}{\frac{1}{2\sqrt{s}} e^{\sqrt{s}}} = \boxed{2\sqrt{s}\left(e^{-\sqrt{s}} - e^{-s-\sqrt{s}}\right)}$

b) $\frac{dx}{dt} = 3t^2 - 12$, $\frac{dy}{dt} = 2t$, $\boxed{\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{2t}{3t^2-12}}$

c) $\frac{dx}{d\omega} = -4\sin(\omega)$, $\frac{dy}{d\omega} = 2\cos(2\omega)$, so $\frac{dy}{dx} = \frac{2\cos(2\omega)}{-4\sin(\omega)} = \boxed{-\frac{1}{2}\frac{\cos(2\omega)}{\sin(\omega)}}$

3) $\dfrac{dy}{dt} = -\sin(t) - \dfrac{1}{t^2}$, $\dfrac{dx}{dt} = 2t + e^t$,

$$\frac{dy}{dx} = \frac{-\sin(t) - \frac{1}{t^2}}{2t + e^t} \qquad \frac{d}{dt}\left(\frac{dy}{dx}\right) = \frac{d}{dt}\left(\left(-\sin(t) - \frac{1}{t^2}\right)(2t+e^t)^{-1}\right) = \left(-\cos(t) + \frac{2}{t^3}\right)(2t+e^t)^{-1}$$
$$+ \left(-\sin(t) - \frac{1}{t^2}\right)(2t+e^t)^{-2}(2+e^t)\Big)$$

So, $$\frac{d^2y}{dx^2} = \frac{\left(-\cos(t) + \frac{2}{t^3}\right)(2t+e^t)^{-1} + \left(-\sin(t) - \frac{1}{t^2}\right)(2t+e^t)^{-2}(2+e^t)}{(2t+e^t)}$$

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4) a) $\dfrac{dx}{dt} = 6t$, $\dfrac{dy}{dt} = 6t^2$, $s = \displaystyle\int_0^1 \sqrt{(6t)^2 + (6t^2)^2}\, dt$

$$= \int_0^1 6t\sqrt{1+t^2}\, dt \qquad u = 1+t^2, \quad du = 2t\,dt$$
$$= \int_1^2 3\sqrt{u}\, du$$
$$= 3 \cdot \frac{2}{3} u^{3/2}\Big|_1^2$$
$$= 2(2^{3/2} - 1).$$

b) $\dfrac{dx}{d\theta} = -4\sin\theta$, $\dfrac{dy}{d\theta} = 4\cos\theta$, $s = \displaystyle\int_0^{2\pi} \sqrt{(4\cos\theta)^2 + (4\sin\theta)^2}\, d\theta$

$$= \int_0^{2\pi} \sqrt{16(\cos^2\theta + \sin^2\theta)}\, d\theta$$
$$= 4\int_0^{2\pi} d\theta$$
$$= 8\pi$$

c) $\dfrac{dx}{dv} = 6v$, $\dfrac{dy}{dv} = 12v^2$, $s = \displaystyle\int_1^3 \sqrt{(6v)^2 + (12v^2)^2}\, dv$

$$= \int_1^3 \sqrt{36v^2(1+4v^2)}\, dv$$
$$= \int_1^3 6v\sqrt{1+4v^2}\, dv \qquad u = 1+4v^2, \quad du = 8v\,dv$$
$$= \frac{6}{8}\int_5^{37} \sqrt{u}\, du$$
$$= \frac{3}{4} \cdot \frac{2}{3} u^{3/2}\Big|_5^{37}$$
$$= \frac{1}{2}\left(37^{3/2} - 5^{3/2}\right)$$